

$$f_1: \mathbb{R}^{1+n} \rightarrow \mathbb{R}, f_2: \mathbb{R}^{1+n} \rightarrow \mathbb{R}, \dots, f_n: \mathbb{R}^{1+n} \rightarrow \mathbb{R}$$

$$f: \mathbb{R}^{1+n} \rightarrow \mathbb{R}^n, \quad f = (f_1, f_2, \dots, f_n)$$

$$(1) \quad y' = f(t, y), \quad y = (y_1, \dots, y_n): \mathbb{R} \rightarrow \mathbb{R}^n.$$

$$y_1' = f_1(t, y_1, \dots, y_n)$$

$$y_2' = f_2(t, y_1, \dots, y_n)$$

$$y_n' = f_n(t, y_1, \dots, y_n)$$

$$(t_0, y_0) = (t_0, y_{01}, y_{02}, \dots, y_{0n}) \in \mathbb{R}^{1+n}$$

$$(2) \quad y_1(t_0) = y_{01}, \dots, y_n(t_0) = y_{0n} \quad y(t_0) = y_0$$

(1), (2) ... Cauchyova úloha

$$\text{Pr.} \quad y_1' = y_1, \quad f_1(\alpha, \beta, \gamma) = \beta$$

$$y_2' = y_1 + t, \quad f_2(\alpha, \beta, \gamma) = \beta + \alpha.$$

$$y(0) = (1, 2).$$

$$y_1' = y_1 \dots y_1(t) = C_1 \cdot e^t, \quad C_1 \in \mathbb{R}$$

$$y_2' = C_1 \cdot e^t + t, \quad y_2(t) = C_1 e^t + \frac{1}{2} t^2 + C_2$$

$$y_1(0) = 1 \dots 1 = C_1 \cdot e^0 \quad C_1 = 1$$

$$y_2(0) = 2 \dots 2 = C_1 \cdot e^0 + \frac{1}{2} 0^2 + C_2 \quad C_2 = 1$$

$$\varphi(t) = (e^t, e^t + \frac{1}{2} t^2 + 1).$$