$F: \mathbb{R}^{n+2} \to \mathbb{R}$ $\quad F(t, y(t), y'(t), \dots, y^{(n)}(t)) = 0$

(1) $F(t, y, y', \dots, y^{(n)}) = 0$

(2) $y^{(n)} = f(t, y, \dots, y^{(n-1)})$, $f: \mathbb{R}^{n+1} \to \mathbb{R}$.

φ je řešením (1) na J $\iff$

$\forall t \in J: F(t, \varphi(t), \varphi'(t), \dots, \varphi^{(n)}(t)) = 0$, $D\varphi = J$.

$(t_0, y_0, \dots, y_{n-1}) \in \mathbb{R}^n$

(3) $y(t_0) = y_0, \; y'(t_0) = y_1, \dots, y^{(n-1)}(t_0) = y_{n-1}$

(1), (3) … Cauchyova úloha

Př. $\quad y'' = e^t$, $\quad y(0) = 0$, $y'(0) = 1$

$\forall t \in (-\infty, \infty)$: $\quad y''(t) = e^t \quad \Rightarrow \quad \exists C_1 \in \mathbb{R}$

$$y'(t) = e^t + C_1 \quad \Rightarrow \quad \exists C_2 \in \mathbb{R}$$

$$y(t) = e^t + C_1 t + C_2 .$$

$y(0) = 0$: $\quad 0 = 1 + C_2 \qquad C_1 = 0$

$y'(0) = \cancel{0}\,1$: $\quad 1 = 1 + C_1 \qquad C_2 = -1$

$$\varphi(t) = e^t - 1.$$

Př. $\quad y'' = y'$, $\quad z = y'$ $\quad z' = z$ $\quad z(t) = C_1 \cdot e^t$, $C_1 \in \mathbb{R}$

$y'(t) = C_1 \cdot e^t$, $\quad y(t) = C_1 \cdot e^t + C_2$, $\quad C_1 \in \mathbb{R} \ni C_2$.