

$$y''' - 2y'' + y' = e^t \quad ; \quad y(0) = 1, y'(0) = 0, y''(0) = 1.$$

$$y(t) = C_1 + C_2 e^t + C_3 \cdot t \cdot e^t + \frac{1}{2} t^2 e^t$$

$$y'(t) = C_2 e^t + C_3 \cdot (t+1) \cdot e^t + (\frac{1}{2} t^2 + t) \cdot e^t$$

$$y''(t) = C_2 e^t + C_3 \cdot (t+2) \cdot e^t + (\frac{1}{2} t^2 + 2t + 1) \cdot e^t$$

$$y(0) = 1 : \quad C_1 + C_2 = 1$$

$$y'(0) = 0 : \quad C_2 + C_3 = 0$$

$$y''(0) = 1 : \quad C_2 + 2C_3 + 1 = 1$$

$$C_2 = -C_3, \quad C_3 = 0, \quad C_2 = 0, \quad C_1 = 1$$

$$y(t) = 1 + \frac{1}{2} t^2 \cdot e^t.$$