

$$y''' - 2y'' + y' = e^t$$

$$\textcircled{A} \quad y''' - 2y'' + y' = 0$$

$$y_h(t) = c_1 + c_2 \cdot e^t + c_3 \cdot t \cdot e^t, \quad c_1 \in \mathbb{R}, c_2 \in \mathbb{R}, c_3 \in \mathbb{R}$$

$$\textcircled{B} \quad y_p(t) = k_1(t) + k_2(t) \cdot e^t + k_3(t) \cdot t \cdot e^t$$

$$\begin{bmatrix} 1 & e^t & t e^t \\ 0 & e^t & (t+1) \cdot e^t \\ 0 & e^t & (t+2) \cdot e^t \end{bmatrix} \cdot \begin{bmatrix} k_1'(t) \\ k_2'(t) \\ k_3'(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ e^t \end{bmatrix}$$

$$k_1'(t) + k_2'(t) \cdot e^t + k_3'(t) \cdot t e^t = 0$$

$$k_2'(t) \cdot e^t + k_3'(t) \cdot (t+1) \cdot e^t = 0$$

$$k_2'(t) \cdot e^t + k_3'(t) \cdot (t+2) \cdot e^t = e^t$$

$$k_3'(t) \cdot e^t = e^t, \quad k_3'(t) = 1$$

$$k_2'(t) = -(t+1), \quad k_1'(t) - (t+1) \cdot e^t + t \cdot e^t = 0$$

$$k_1'(t) = e^t, \quad k_1(t) = e^t, \quad k_2(t) = -\left(\frac{t^2}{2} + t\right), \quad k_3(t) = t.$$

$$y_p(t) = e^t - \left(\frac{t^2}{2} + t\right) \cdot e^t + t^2 \cdot e^t \Rightarrow y_p(t) = e^t \cdot (1-t) + \frac{t^2}{2} \cdot e^t$$

$$\textcircled{C} \quad Y \dots y(t) = c_1 + c_2 \cdot e^t + c_3 \cdot t \cdot e^t + e^t(1-t) + \frac{t^2}{2} \cdot e^t.$$