(1) $y' + y = \cos t$

(A) $y' + y = 0$ $\quad Y_H \dots C \cdot e^{-t}, \; C \in \mathbb{R}$

(B) $y_p(t) = k(t) \cdot e^{-t}$

$y_p'(t) = k'(t) \cdot e^{-t} + k(t) \cdot e^{-t} \cdot (-1)$

$k'(t) \cdot e^{-t} \cancel{- k(t) \cdot e^{-t}} \cancel{+ k(t) \cdot e^{-t}} = \cos t$

$k'(t) \cdot e^{-t} = \cos t \; , \quad k'(t) = \cos t \cdot e^{t}$

$k(t) = \int \cos t \cdot e^{t} dt = \cos t \cdot e^{t} - \int (-\sin t) \cdot e^{t} dt =$

$\left| \begin{matrix} u = \cos t & v' = e^{t} \\ u' = -\sin t & v = e^{t} \end{matrix} \right| \quad = \cos t \cdot e^{t} + \int \sin t \cdot e^{t} dt$

$\left| \begin{matrix} u = \sin t & v' = e^{t} \\ u' = \cos t & v = e^{t} \end{matrix} \right| . \quad = \cos t \cdot e^{t} + \sin t \cdot e^{t} - \underbrace{\int \cos t \cdot e^{t} dt}_{k(t)}$

$k(t) = \cos t \, e^{t} + \sin t \, e^{t} - k(t) \quad \Rightarrow$

$k(t) = \frac{e^{t}}{2} (\cos t + \sin t) \quad \Rightarrow$

$y_p(t) = \frac{1}{2} (\cos t + \sin t)$

(C) $Y \dots \underline{\underline{y(t) = C \cdot e^{-t} + \frac{1}{2}(\cos t + \sin t), \; C \in \mathbb{R}}}$