(1) $y' + y = \cos t$

$y' + a(t)y = b(t)$

$a(t) = 1,\ a \in C(\mathbb{R})$; $b(t) = \cos t,\ b \in C(\mathbb{R})$

Metoda přenásobení

$$e^{\int a(t)dt} = e^{\int 1 dt} = e^{t}$$

$$y' \cdot e^t + y \cdot e^t = \cos t \cdot e^t$$

$$(y \cdot e^t)' = \cos t \cdot e^t$$

(*) $$y \cdot e^t = \int \cos t \cdot e^t dt + C,\ C \in \mathbb{R}$$

$$I = \int \cos t \cdot e^t dt = \cos t \cdot e^t - \int (-\sin t\, e^t) dt =$$

$\left| \begin{matrix} u = \cos t & v' = e^t \\ u' = -\sin t & v = e^t \end{matrix} \right|$ $= \cos t\, e^t + \int \sin t\, e^t dt =$

$\left| \begin{matrix} u = \sin t & v' = e^t \\ u' = \cos t & v = e^t \end{matrix} \right|$ $= \cos t \cdot e^t + \sin t\, e^t - \underbrace{\int \cos t\, e^t dt}_{I}$

$$I = \cos t\, e^t + \sin t\, e^t - I \Rightarrow I = \frac{e^t}{2}(\cos t + \sin t)$$

$$y \cdot e^t = \frac{e^t}{2} \cdot (\cos t + \sin t) + C \Rightarrow$$

$$y = C \cdot e^{-t} + \frac{1}{2}(\cos t + \sin t),\ C \in \mathbb{R}$$